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Asymptotic symmetries of string theory on $AdS_3 \times S^3$ with Ramond-Ramond fluxes

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Abstract: String theory on AdS_3 space-times with boundary conditions that allow for black hole states has global asymptotic symmetries which include an infinite dimensional conformal algebra. Using the conformal current algebra for sigma-models on $PSU(1,1|2)$, we explicitly construct the R-symmetry and Virasoro charges in the worldsheet theory describing string theory on $AdS_3 \times S^3$ with Ramond-Ramond fluxes. We also indicate how to construct the full boundary superconformal algebra. The boundary superconformal algebra plays an important role in classifying the full spectrum of string theory on AdS_3 with Ramond-Ramond fluxes, and in the microscopic entropy counting in D1-D5 systems.

Keywords: AdS-CFT Correspondence, Conformal Field Models in String Theory

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1 Introduction

The holographic correspondence between gauge theories and gravitational theories has shed light both on non-perturbative quantum gravity and on strong coupling phenomena in gauge theory. The quintessential example of a pair of theories related by holography is the four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory, and type IIB string theory on the $AdS_5 \times S^5$ space-time with Ramond-Ramond flux [1]. The further development of the correspondence has been somewhat hampered by the difficulty in solving string theory on non-trivially curved backgrounds with Ramond-Ramond flux (except in a plane wave limit [2, 3]). A tool that has stimulated significant progress is the integrability of the spectrum of the dilatation operator in $\mathcal{N} = 4$ super Yang-Mills theory [4] as well as the integrability of the bulk worldsheet σ-model [5, 6]. The integrable structure is coded in the existence of an infinite number of non-local charges.

In a particular instance of the holographic correspondence, one can make headway via the existence of an infinite number of *local* charges. Indeed $AdS_3 \times S^3$ bulk superstring theory is dual to a conformal field theory in two dimensions. It therefore has the exceptional property of having an infinite number of local charges which extend the finite dimensional isometry group [7]. We believe it is important to construct the symmetry algebra in the bulk string theory explicitly and to exploit it maximally in classifying the spectrum. It will moreover be interesting to see how the local infinite dimensional symmetry algebra intertwines with the integrability of the model.

Quantum gravity on AdS_3 space-times supplemented with boundary conditions that allow for black hole solutions has an asymptotic symmetry group which includes the two-dimensional conformal algebra [7]. For string theory on an AdS_3 background with Neveu-Schwarz-Neveu-Schwarz flux, the space-time symmetry generators were explicitly constructed in terms of worldsheet operators in [8–10].

Here we concentrate on $AdS_3 \times S^3$ backgrounds of string theory with non-zero Ramond-Ramond flux with eight or sixteen supercharges. These backgrounds arise as near-brane geometries of D1-D5 brane configurations (which may also include fundamental strings and NS5-branes). After introducing a third charge, the conformal symmetry is central in the microscopic counting that reproduces the Bekenstein-Hawking area formula for the black hole entropy. We believe it is useful to exhibit this symmetry directly and explicitly in the D1-D5 near-brane geometry.

To that end we work in the hybrid formalism of [11] which renders eight spacetime supercharges manifest. In that formalism the central part of the worldsheet model is the sigma-model on the $PSU(1,1|2)$ supergroup (or rather, its universal cover), which contains a bosonic $AdS_3 \times S^3$ subspace. The main building block in constructing the vertex operators of the spacetime symmetry generators are the curents of the $PSU(1,1|2)$ supergroup model which satisfy a conformal current algebra found in [12]. Technically, our analysis is a non-chiral version of the construction of the super Virasoro algebra in [9, 10, 13] for the case of F1-NS5 backgrounds. For interesting studies of the supergroup sigma-model on $PSU(1,1|2)$ at the Wess-Zumino-Witten points and beyond in the context of string theory we refer to [14–17].

This article is organized as follows. In section 2 we review key features of the worldsheet sigma-model in the hybrid formalism. The worldsheet current algebra of the model is recalled in section 3. The construction of the vertex operators for the R-current algebra generators in space-time and the calculation of their operator product expansion (OPE) is performed in section 4. In section 5 we indicate how to construct the full space-time super Virasoro algebra and discuss the properties of the operator that plays the role of

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the central extension in the spacetime algebra. We summarize and make our concluding remarks in section 6. Some technical details regarding the worldsheet action and certain aspects regarding worldsheet operators and their OPEs are collected in the appendices.

2 String theory on $AdS_3 \times S^3$ with Ramond-Ramond fluxes

In this section, we briefly review the hybrid formalism, the supergroup sigma-model that describes the $AdS_3 \times S^3$ background with Ramond-Ramond fluxes, and its relation to the near-brane geometries. We also discuss the BRST operator of the worldsheet theory in the hybrid formalism, whose cohomology determines the physical string Hilbert space.

2.1 The hybrid formalism

The hybrid formalism introduced in [18] allows one to covariantly quantize string theory with Ramond-Ramond fluxes in a six-dimensional space-time. It renders eight supercharges in space-time manifest.[1] The formalism is based on defining space-time fermions $\theta^{a\alpha}$ and their conjugate momenta $p_{a\alpha}$ in terms of the spin-fields in the RNS formalism. There are six bosons corresponding to the six space-time directions as well as two chiral interacting bosons ρ and σ (and their right-moving counterparts $\bar{\rho}$ and $\bar{\sigma}$) related to the bosonized ghost systems of the RNS formalism.

In the $AdS_3 \times S^3$ background the ghost fields $\phi = -\rho - i\sigma$ and $\bar{\phi} = -\bar{\rho} - i\bar{\sigma}$ are respectively chiral and anti-chiral (up to interaction terms). The ghosts appear in the action as exponentials of the fields ϕ and $\bar{\phi}$ with positive exponent. In other words, the worldsheet action is a perturbation series in the variables e^{ϕ} and $e^{\tilde{\phi}}$. In the presence of non-zero Ramond-Ramond flux, the worldsheet Lagrangian is quadratic in the fermionic momenta $p_{a\alpha}$. It is then possible to integrate out the fermionic momenta, so that the action only depends on the bosonic and fermionic coordinates (as well as the ghosts) [11].

2.2 The supergroup sigma model

We will work to lowest order in the ghost exponentials. At this order, the action pertaining to the six-dimensional space $AdS_3 \times S^3$ is a non-linear sigma-model with target space the (covering of the) supergroup manifold $PSU(1,1|2)$ [11]. The supergroup $PSU(1,1|2)$ has a maximal bosonic subgroup which is $\mathrm{SU}(1,1) \times \mathrm{SU}(2)$. There is a corresponding $psu(1,1|2)$ superalgebra which is a particular real form of the $psl(2|2)$ superalgebra. The latter is defined by its generators and their (anti-)commutation relations are given by

$$\begin{aligned} [K_{ab}, K_{cd}] &= \delta_{ac} K_{bd} - \delta_{ad} K_{bc} - \delta_{bc} K_{ad} + \delta_{bd} K_{ac} \\ [K_{ab}, S_{c\alpha}] &= \delta_{ac} S_{b\alpha} - \delta_{bc} S_{a\alpha} \\ \{S_{a\alpha}, S_{b\beta}\} &= \frac{1}{2} \epsilon_{\alpha\beta} \epsilon_{abcd} K^{cd} \,. \end{aligned} \tag{2.1}$$

[1] There exists another formulation of the six-dimensional hybrid string which renders sixteen supercharges manifest [19]. In this formalism, the space-time $AdS_3 \times S^3$ is embedded in a super-coset. We concentrate on the formalism defined in [18] in which the manifold $AdS_3 \times S^3$ is embedded in a supergroup [11].

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where a, b are vector indices of $so(4) \sim sl(2) \times sl(2)$. The generators $K_{ab} = -K_{ba}$ are the bosonic generators. There is an outer automorphism algebra $sl(2)_{\text{out}}$ (which has the real form $su(2)_{\text{out}}$ in the case of the algebra $psu(1,1|2)$) and the indices α, β run over the states in a doublet of $sl(2)_{\text{out}}$. The tensor $\epsilon_{\alpha\beta}$ is anti-symmetric. Under the bosonic subalgebra and the outer automorphism algebra $sl(2) \oplus sl(2) \oplus sl(2)_{\text{out}}$ the six bosonic generators transform as $(3,1,1) + (1,3,1)$ and the eight fermionic generators as $(2,2,2)$ for a total of fourteen generators that span the adjoint of $psl(2|2)$.

There is no fundamental representation of the $psl(2|2)$ algebra, but in the appendix we give an explicit 4×4 matrix parameterization of the $sl(2|2)$ algebra (which is the $psl(2|2)$ algebra augmented with a central bosonic generator). Explicit calculations can be performed with the matrix parameterization of this algebra, and results for the $psl(2|2)$ model are obtained by dividing out by the central generator.

The action of the non-linear sigma-model on the supergroup is:

$$\begin{aligned} S &= S_{\text{kin}} + S_{WZ} \\ S_{\text{kin}} &= \frac{1}{16\pi f^2} \int d^2z Tr'[-\partial^\mu g^{-1} \partial_\mu g] \\ S_{WZ} &= -\frac{ik}{24\pi} \int_B d^3y \epsilon^{\alpha\beta\gamma} Tr'(g^{-1}\partial_\alpha g g^{-1} \partial_\beta g g^{-1} \partial_\gamma g) \end{aligned} \tag{2.2}$$

where g takes values in the supergroup $PSU(1,1|2)$ and Tr' indicates the non-degenerate bi-invariant metric. It can be thought of as the supertrace in the $su(1,1|2)$ superalgebra. We parameterize the group element $g \in \mathrm{SU}(1,1|2)$ as

$$g = e^\alpha e^{\theta^{a\alpha} S_{a\alpha}} g_{S^3} g_{AdS_3} , \tag{2.3}$$

where the first factor represents the U(1) to be divided out, the second factor the fermions, the third one an element of the group SU(2) and the last factor an element of the group $\mathrm{SL}(2,\mathbb{R})$. We spell out the non-linear supergroup sigma-model action in much more detail in terms of a global coordinate system in appendix A.2. From the kinetic term in equation (A.35), we find that the quadratic fermionic term in the action takes the form

$$S_{\text{fermionic}} = \frac{1}{4\pi f^2} \int d^2z \; \delta_{ab} \epsilon_{\alpha\beta} \partial\theta^{a\alpha} \bar{\partial}\theta^{b\beta} + \dots \tag{2.4}$$

The main advantage of the hybrid formalism compared to the Green-Schwarz superstring is that it is covariantly quantizable.

2.3 The brane configuration

The coefficient $\frac{1}{f^2}$ multiplying the kinetic term is the square of the spacetime radius. The coefficient k multiplying the Wess-Zumino term is quantized. It is equal to the number of units of Neveu-Schwarz-Neveu-Schwarz flux on the three-sphere. When $AdS_3 \times S^3$ is realized as the near-brane geometry of a system of NS5-F1-D5-D1 branes, the couplings $\frac{1}{f^2}$ and k are related to the number of NS5-branes Q_{NS5} and D5-branes Q_{D5} through the

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formulas [11]:

$$\frac{1}{f^2} = \sqrt{Q_{NS5}^2 + g_s^2 Q_{D5}^2}$$
$$k = Q_{NS5}\,, \tag{2.5}$$

where g_s is the ten-dimensional string coupling constant. Note that since the D5-branes are a factor of $1/g_s$ lighter than the NS5-branes, they curve the geometry less strongly. Supersymmetry requires the relative number of D1-branes compared to F1-strings to be equal to the relative number of D5-branes compared to NS5-branes. We thus have:

$$(Q_{NS5}, Q_{D5}) = Q_5(p,q) \quad \text{and} \quad (Q_{F1}, Q_{D1}) = Q_1(p,q)\,. \tag{2.6}$$

and via the attractor mechanism Q_1 (divided by Q_5) fixes the volume of the compactification manifold. For a type IIB superstring background with sixteen or eight supercharges, the internal manifold can be either a four-torus or a K3 manifold.

2.4 The physical Hilbert space

The worldsheet theory also contains an $N = 4$ superconformal model at central charge $c = 6$ associated to the compactification manifold. Via the ghosts, the principal chiral model with Wess-Zumino term is coupled to the compact theory. The correlation functions of the model can be defined in terms of the prescription for computing $N = 4$ topological string amplitudes [18].

The physical string Hilbert space is given in terms of a set of constraints that can be defined in terms of an $N = 2$ superconformal algebra. In particular, physical states belong to the cohomology of the charges associated to the BRST currents G^+ and $\bar{G}^+$. At zeroth order in the ghost exponential e^ϕ, the holomorphic BRST current G^+ reads:

$$G^+ = e^{i\sigma}\left(T_{PSU(1,1|2)} + \frac{1}{2}(\partial\phi\partial\phi + \partial^2\phi)\right) + G_C^+ + \mathcal{O}(e^\phi) \tag{2.7}$$

where G_C^+ depends only on the compactification variables and the ghosts, and $T_{PSU(1,1|2)}$ is the holomorphic stress-energy tensor of the supergroup sigma-model, which is given in terms of a generalized Sugawara construction [12].

Our goal is to construct, in the worldsheet theory, the vertex operators for the generators of the asymptotic symmetry group for the $AdS_3 \times S^3$ string theory. We will show that these vertex operators are BRST closed with respect to the charge associated to the current (2.7). As such, we will be able to insert these vertex operators in correlation functions to generate space-time Ward-identities.

3 The worldsheet conformal current algebra

In the previous section we reviewed that the supergroup sigma-model on $PSU(1,1|2)$ is the central building block for $AdS_3 \times S^3$ string theory with Ramond-Ramond flux in the hybrid formalism. The supergroup sigma-model has zero Killing form and it therefore falls into

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the class of models for which the worldsheet conformal current algebra was analyzed in [12]. Since the worldsheet current algebra will be the central technical tool in constructing the space-time Virasoro algebra, we review it here. We also introduce current algebra primary fields and some useful notations.

3.1 Conformal current algebra

From the action (2.2) we can calculate the classical currents associated to the invariance of the theory under left multiplication of the field g by a group element in G_L and right multiplication by a group element in G_R. The classical G_L currents are given by

$$\begin{aligned} j_{L,z} &= c_+ \partial g g^{-1} \\ j_{L,\bar{z}} &= c_- \bar{\partial} g g^{-1}\,, \end{aligned} \tag{3.1}$$

where the constant c_+ and c_- are given in terms of the couplings by:

$$c_\pm = -\frac{(1 \pm k f^2)}{2f^2}\,. \tag{3.2}$$

Similarly, we also have the left-invariant currents that generate right multiplication:

$$\begin{aligned} j_{R,z} &= -c_- g^{-1} \partial g \\ j_{R,\bar{z}} &= -c_+ g^{-1} \bar{\partial} g\,. \end{aligned} \tag{3.3}$$

The operator product expansions satisfied by the left currents have been derived in [12], where now, a denotes a super Lie algebra valued index:

$$\begin{aligned} j^a_{L,z}(z) j^b_{L,z}(0) &\sim \kappa^{ab} \frac{c_1}{z^2} + f^{ab}{}_c \left[\frac{c_2}{z} j^c_{L,z}(0) + (c_2 - g) \frac{\bar{z}}{z^2} j^c_{L,\bar{z}}(0) \right] + \dots \\ j^a_{L,\bar{z}}(z) j^b_{L,\bar{z}}(0) &\sim \kappa^{ab} \frac{c_3}{\bar{z}^2} + f^{ab}{}_c \left[\frac{c_4}{\bar{z}} j^c_{L,\bar{z}}(0) + (c_4 - g) \frac{z}{\bar{z}^2} j^c_{L,z}(0) \right] + \dots \\ j^a_{L,z}(z) j^b_{L,\bar{z}}(0) &\sim \tilde{c} \kappa^{ab} 2\pi \delta^{(2)}(z-w) + f^{ab}{}_c \left[\frac{(c_4 - g)}{\bar{z}} j^c_{L,z}(0) + \frac{(c_2 - g)}{z} j^c_{L,\bar{z}}(0) \right] + \dots \end{aligned} \tag{3.4}$$

The ellipses refer to subleading terms proportional to the derivatives of the current. We will only need the leading singular behaviour of the operator product expansions to derive the spacetime superconformal algebra. The right current components $j_{R,z}$ and $j_{R,\bar{z}}$ satisfy similar operator product expansions with the holomorphic coordinates replaced by anti-holomorphic ones. In appendix A.2 we give explicit expressions for left-invariant right-moving currents in the $PSU(1,1|2)$ supergroup model in a particular coordinate system. For the supergroup non-linear sigma-model in equation (2.2), the coefficients of the conformal current algebra, expressed purely in terms of $c_\pm$, are given by [12]

$$\begin{aligned} c_1 &= -\frac{c_+^2}{c_+ + c_-} & c_3 &= -\frac{c_-^2}{c_+ + c_-} \\ c_2 &= i\frac{c_+(c_+ + 2c_-)}{(c_+ + c_-)^2} & c_4 &= i\frac{c_-(2c_+ + c_-)}{(c_+ + c_-)^2} \\ g &= i\frac{2c_+ c_-}{(c_+ + c_-)^2} & \tilde{c} &= \frac{c_+ c_-}{c_+ + c_-}\,, \end{aligned} \tag{3.5}$$

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where $c_\pm$ are the factors defined in (3.2) that normalize the currents. For future purposes, we note that due to the existence of the elementary group valued field g these coefficients satisfy the equations[2]

$$\frac{\tilde{c}-c_1}{c_+} = 1 = \frac{\tilde{c}-c_3}{c_-} \qquad \text{and} \qquad c_2+c_4-g=i. \tag{3.6}$$

3.2 The $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ global symmetry

The zero-mode of the global current

$$\mathcal{J}^a_L = -i(j^a_{L,z} - j^a_{L,\bar{z}}) \tag{3.7}$$

generates the symmetry which is the left translation of a supergroup element by an element of the supergroup. Restricting the index a to just the $\mathrm{SL}(2,\mathbb{R})$ directions, we obtain the generators of the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ isometry group of AdS_3, which is a subgroup of the left/right space-time Virasoro algebra. In holography, the AdS_3 submanifold of the supergroup plays a special role, which makes it useful to decompose observables in the spacetime theory in terms of representations of this subgroup. We introduce auxiliary complex variables $(x,\bar{x})$ in terms of which the global $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ generators are expressed simply in terms of differential operators.

A given observable $O(x,\bar{x})$ in an irreducible representation of the group, can be generated from its value at $x=\bar{x}=0$ by acting with the lowering generators of the $sl(2,\mathbb{R})$ algebra $\mathcal{J}^-_{L,R}$

$$\mathcal{O}(x,\bar{x}) = e^{-x\mathcal{J}^-_{L,0}-\bar{x}\mathcal{J}^-_{R,0}}\mathcal{O}(0,0)\,, \tag{3.8}$$

where the 0 index indicates the zero mode of the current. In particular, we can apply this operation to a current component in the adjoint representation of the group:

$$\begin{aligned} j_{L,z}(x;z) \equiv j^+_{L,z}(x,;z,\bar{z}) &= e^{-x\mathcal{J}^-_{L,0}} j^+_{L,z}(z,\bar{z}) e^{x\mathcal{J}^-_{L,0}} \\ &= j^+_{L,z}(z,\bar{z}) - 2x j^3_{L,z}(z,\bar{z}) + x^2 j^-_{L,z}(z,\bar{z})\,. \end{aligned} \tag{3.9}$$

Similar relations hold for the $\bar{z}$ component as well as the right-currents. We have for instance:

$$j_{R,\bar{z}}(\bar{x};z) = j^+_{R,\bar{z}}(z,\bar{z}) - 2\bar{x} j^3_{R,\bar{z}}(z,\bar{z}) + \bar{x}^2 j^-_{R,z}(z,\bar{z})\,. \tag{3.10}$$

In space-time the $+$ component of the currents j^+_L is of conformal weight $(-1,0)$ with respect to the zero-modes of the Virasoro algebras.

[2]Roughly speaking, we can think of $\log g$ as having a logarithmic operator product expansion with itself, as in an abelian theory. The current component operator product expansions are then derivatives of this more basic logarithmic OPE. That gives rise to relations between c_1, c_3 and $\tilde{c}$ (which leads to the first two identities, in a particular normalization for the currents). The Ward identity for the left translation of the group valued field g gives rise to the third identity.

3.3 Left current algebra primaries

It will be useful to define fields which are primaries with respect to the left current algebra. A left primary field ϕ with respect to the current algebra (3.4) is a field satisfying the operator product expansions:

$$\begin{aligned} j^a_{L,z}(z,\bar{z})\phi(w,\bar{w}) &= -\frac{c_+}{c_+ + c_-} t^a \frac{\phi(w,\bar{w})}{z-w} + \text{less singular} \\ j^a_{L,\bar{z}}(z,\bar{z})\phi(w,\bar{w}) &= -\frac{c_-}{c_+ + c_-} t^a \frac{\phi(w,\bar{w})}{\bar{z}-\bar{w}} + \text{less singular} \end{aligned} \tag{3.11}$$

where the matrices t^a are the generators of the Lie super-algebra taken in the representation in which ϕ transforms. The coefficients are fixed by the global transformation properties of the field and the demand that the field ϕ have trivial operator product expansion with the Maurer-Cartan operator $c_+\partial j^a_{\bar{z}} - c_-\bar{\partial} j^a_z + \frac{i}{2} f^a{}_{bc}(: j^c_z j^b_{\bar{z}} : + (-1)^{bc} : j^b_{\bar{z}} j^c_z :)$.

3.4 Representations of the global bosonic symmetry group

For later purposes, we define a bosonic field Φ_h transforming in the representation $(\mathcal{D}^+_h, 0)$ of the bosonic subgroup $SL(2,\mathbb{R}) \times SU(2)$ of the supergroup.[3] When we continue AdS_3 to Euclidean signature it will function as the unique bulk-to-boundary propagator for a scalar field coupling to a space-time operator of dimension h. We think of the bosonic field $\Phi_h(y,\bar{y};w,\bar{w})$ as parameterizing (via the variables $y,\bar{y}$) a subspace of a representation of the supergroup corresponding to a left primary field, in which case the operator product expansions of this field with the components of the current $j_L(x;z,\bar{z})$ defined in equation (3.9) read:

$$\begin{aligned} j_{L,z}(x;z,\bar{z})\Phi_h(y,\bar{y};w,\bar{w}) &= \frac{c_+}{c_+ + c_-}\frac{1}{z-w}[(y-x)^2\partial_y + 2h(y-x)]\Phi_h \\ j_{L,\bar{z}}(x;z,\bar{z})\Phi_h(y,\bar{y};w,\bar{w}) &= \frac{c_-}{c_+ + c_-}\frac{1}{\bar{z}-\bar{w}}[(y-x)^2\partial_y + 2h(y-x)]\Phi_h. \end{aligned} \tag{3.12}$$

We have used the fact that the generators t^a in the representation in which Φ_h transform can be written as the following differential operators:

$$t^3 = -x\partial_x - h\,; \quad t^+ = -x^2\partial_x - 2hx\,; \quad t^- = -\partial_x\,, \tag{3.13}$$

in the conventions of [10]. It is also useful to rewrite the operator product expansion of the $SL(2,\mathbb{R})$ components of the left-currents as follows:

$$\begin{aligned} j_{L,z}(x;z)\cdot j_{L,z}(y;w) &= \frac{c_1(x-y)^2}{(z-w)^2} + \frac{c_2\, f^{ab}{}_c}{i(z-w)}\left[(y-x)^2\partial_y - 2(y-x)\right] j_{L,z} \\ &\quad + \frac{(c_2-g)\,(\bar{z}-\bar{w}) f^{ab}{}_c}{i(z-w)^2}\left[(y-x)^2\partial_y - 2(y-x)\right] j_{L,\bar{z}} + \ldots \end{aligned} \tag{3.14}$$

After these worldsheet preliminaries, we turn to the construction of the space-time R-current.

[3] In our conventions, the quadratic casimir of the representation $\mathcal{D}^+_h$ of $SL(2,\mathbb{R})$ is $h(h-1)$.

4 The spacetime R-current

4.1 R-symmetry generators from non-trivial diffeomorphisms

We consider the space-time background $AdS_3 \times S^3 \times X$ which is a solution of type IIB string theory. The compact space X can be either T^4 or $K3$. In the following discussion we will focus on the $AdS_3 \times S^3$ factor. The massless excitations of the string give the supergravity multiplet and one tensor multiplet of $D = 6$, $N = 2$ supergravity. In the hybrid formalism, this has been shown in detail in [11, 20] by computing the cohomology of the BRST operator associated to the current in equation (2.7). If we further reduce the theory down to AdS_3, the fluctuations of the metric with one index in AdS_3 and one index in S^3 give rise to an SU(2)-valued massless vector field in three dimensions. These gauge bosons in the bulk are associated with dimension one currents on the boundary. The space-time conformal field theory exhibits, in particular, an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ R-symmetry group corresponding to the bulk isometries of the three-sphere.

The R-symmetry algebra extends to a current algebra on the boundary. From the bulk point of view, this is due to the existence of diffeomorphisms which act non-trivially on the space of bulk configurations with particular boundary conditions. Diffeomorphisms that fall off slowly generate a non-trivial asymptotic symmetry group. For the supersymmetric backgrounds under consideration, these diffeomorphisms were analyzed in [21, 22] in the supergravity approximation.

Diffeomorphisms that induce a non-vanishing transformation at infinity act non-trivially on the Hilbert space of the theory. We parameterize AdS_3 with the Gaussian coordinate system $(\gamma, \bar{\gamma}, \phi)$ on $\mathrm{SL}(2,\mathbb{R})$ which admits an analytic continuation to euclidean AdS_3 in Poincaré coordinates. In these coordinates the AdS_3 metric takes the form:

$$ds^2_{AdS_3} = d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma}. \tag{4.1}$$

The asymptotic symmetry group is generated by diffeomorphisms for which the generating vector field ξ behaves near the boundary as

$$\xi^\mu = f(\gamma, \bar{\gamma}) + \mathcal{O}(e^{-\phi}), \tag{4.2}$$

where the index μ, runs over the six indices corresponding to the AdS_3 and S^3 directions. We can further expand the function $f(\gamma, \bar{\gamma})$ in powers of γ and $\bar{\gamma}$. Since the coordinates $(\gamma, \bar{\gamma})$ parameterize radial slices, this is equivalent to a Fourier mode expansion in the boundary coordinates. The integrated string vertex operator for an SU(2) gauge boson is[4]

$$\int d^2 z \left(j^a_{L,z} j^m_{L,\bar{z}} + j^a_{L,\bar{z}} j^m_{L,z} \right) g_{am} , \tag{4.3}$$

where, from now on, we specify a to be the SU(2) index, m is an $\mathrm{SL}(2,\mathbb{R})$ index, and $g_{\mu\nu}$ is the six-dimensional metric. Under a diffeomorphism generated by the vector field ξ^μ, the

[4] Whether we write down the vertex operator in terms of the left- or of the right-current is a matter of convention. Indeed both one-forms associated to the left- and right-currents generate a suitable basis for the cotangent bundle in spacetime.

metric changes as $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}$. So the integrated vertex operator that generates a gauge transformation for the gauge boson is

$$\int d^2z \left(j^a_{L,z} j^m_{L,\bar{z}} + j^a_{L,\bar{z}} j^m_{L,z} \right) \partial_m \xi_a \,. \tag{4.4}$$

The vector field ξ_a has an SU(2) index, and depends only on the AdS_3 coordinates. Such a vertex operator can be BRST non-trivial if the vector field ξ does not vanish fast enough at infinity. This is related to the fact that a state of the form $Q_{\rm BRST}|\phi\rangle$ is not BRST exact if the state $|\phi\rangle$ does not belong to the Hilbert space. Working at first order in the fermionic currents, we can rewrite the vertex operator as:

$$\int d^2z (j^a_{L,z} \bar{\partial}\xi_a + j^a_{L,\bar{z}} \partial \xi_a). \tag{4.5}$$

We define the n-th mode of the boundary R-current J^a_n as:

$$J^a_n = \int d^2z (j^a_{L,z} \bar{\partial}\xi^{(n)} + j^a_{L,\bar{z}} \partial \xi^{(n)}) \,, \tag{4.6}$$

where $\xi^{(n)} = \gamma^n + \mathcal{O}(e^{-\phi})$ near the boundary at $\phi \to \infty$. We define the left-moving boundary R-current $J^a(x)$ as:

$$J^a(x) = \sum_{n=-\infty}^{\infty} \frac{J^a_n}{x^{n+1}} \tag{4.7}$$

where we have introduced the variable x that parameterizes the eigenvalue of the parabolic generator of the symmetry group $SL(2,\mathbb{R})_L$. Asymptotically, the left-moving R-current therefore has the form

$$J^a(x) = \int d^2z \left[j^a_{L,z} \bar{\partial} \left(\frac{1}{\gamma - x} + \mathcal{O}(e^{-\phi}) \right) + j^a_{L,\bar{z}} \partial \left(\frac{1}{\gamma - x} + \mathcal{O}(e^{-\phi}) \right) \right] . \tag{4.8}$$

Similarly, we can also introduce the variable $\bar{x}$ related to $SL(2,\mathbb{R})_R$. We can then define $\bar{J}^a(\bar{x})$ using diffeomorphisms with $\xi_{(n)} = \bar{\gamma}^n + \mathcal{O}(e^{-\phi})$. The variables $(x, \bar{x})$ can also be interpreted to parameterize the manifold on which the spacetime two-dimensional conformal field theory is defined.

Applying the diffeomorphism transformation to the action written out in detail in appendix A, one can derive the explicit expression for the R-current in equation (4.8).

Non-trivial diffeomorphisms for the interacting theory. In order to write down the exact expression for the R-current $J^a(x)$ in a convenient way we define the function $\Lambda(x, \bar{x}; \gamma, \bar{\gamma}, \phi)$, first introduced in [10]:

$$\Lambda(x, \bar{x}; \gamma, \bar{\gamma}, \phi) = -\frac{1}{\gamma - x} \left[\frac{(\gamma - x)(\bar{\gamma} - \bar{x})e^{2\phi}}{1 + (\gamma - x)(\bar{\gamma} - \bar{x})e^{2\phi}} \right] \tag{4.9}$$

We propose the following expression for the boundary R-current in the fully interacting theory:

$$J^a(x) = \frac{1}{\pi} \int d^2z (j^a_{L,z} \bar{\partial}\Lambda(x, \bar{x}; z, \bar{z}) + j^a_{L,\bar{z}} \partial \Lambda(x, \bar{x}; z, \bar{z})) \,. \tag{4.10}$$

Since Λ behaves near the boundary as $\Lambda \approx -\frac{1}{\gamma - x}$, it coincides with the expression derived in the weak coupling, near-boundary region in (4.8). The expression we put forward for the R-current is the natural non-chiral generalization of the expressions of [10] for the vertex operators which generate the asymptotic symmetry algebra. Notice that a different choice for the parameter $\xi^{(n)}$ would produce an operator related to $J^a(x)$ by a trivial gauge transformation. The advantage of our choice of subleading behaviour is that the function Λ has a simple behaviour under the action of the global symmetry group of the theory. Indeed, the function Λ satisfies the equation

$$\partial_{\bar{x}} \Lambda = \pi \Phi_1 \,, \tag{4.11}$$

where the function Φ_1 is defined as:

$$\Phi_1(x, \bar{x}; \gamma, \bar{\gamma}, \phi) = \frac{1}{\pi} \left(\frac{1}{(\gamma - x)(\bar{\gamma} - \bar{x}) e^{\phi} + e^{-\phi}} \right)^2 . \tag{4.12}$$

This function is an eigenvector of the laplacian operator on euclidean AdS_3, with zero eigenvalue. Near the boundary, this wave function behaves like a delta-function identifying γ and x. It is thus the bulk-to-boundary propagator of a massless scalar from the boundary point $(x, \bar{x})$ to the bulk point $(\gamma, \bar{\gamma}, \phi)$.

In the worldsheet theory Φ_1 is a primary field with respect to the current algebra. It transforms in the discrete $\mathcal{D}_1^+ \times \mathcal{D}_1^+$ representation of the $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SL}(2,\mathbb{R})_R$ current algebra and with spin zero under the action of the bosonic subgroup $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. We can extend the representation to a representation of the full supergroup.

We wish to argue that the following equation holds true:

$$\lim_{z \to w} \Phi_1(x, \bar{x}; z, \bar{z}) \Phi_1(y, \bar{y}; w, \bar{w}) = \delta^{(2)}(x - y) \Phi_1(y, \bar{y}; w, \bar{w}) + \mathcal{O}(z - w, \bar{z} - \bar{w}) \tag{4.13}$$

The first justification for this is the matching of the worldsheet and space-time conformal dimensions of the left and right hand sides. Moreover, we recall that in the case of the Wess-Zumino-Witten model, this equation was argued for on semi-classical grounds in [10], and was made precise in [23]. In [23] the three-point function for fields in the continuous representations in the Wess-Zumino-Witten model were analytically continued to discrete values of the spin. In doing so, one picks up residues of poles when shifting the contour of integration over the spins that appear in the product of two operators in the continuous representation. These poles can arise from the dynamical three-point function or from $\mathrm{SL}(2,\mathbb{R})$ representation theory. What is important to us is that the delta-function in equation (4.13) arose from $\mathrm{SL}(2,\mathbb{R})$ group theoretic properties, namely from the analytic continuation in the spin j of the Clebsch-Gordan coefficient of the form $|x|^{2j}$. Since this part of the three-point function is universal, it is natural to assume that the delta-function appearing in the product above is universal as well.

We collect some important formulae below that will prove to be very useful in our later calculations. We begin by noting that for $h = 1$, equation (3.12) reads

$$j_{L,z}(x; z) \Phi_1(y, \bar{y}; w, \bar{w}) = \frac{1}{z - w} \partial_y [(x - y)^2 \Phi_1(y, \bar{y}; w, \bar{w})] \tag{4.14}$$

Moreover equation (B.7) reads, using the expression (3.13) for the $SL(2,\mathbb{R})$ generators in terms of differential operators:

$$\partial_z \Phi_1(x,\bar{x};z,\bar{z}) = -\frac{1}{c_+}\partial_x[: j_{L,z}\Phi_1 : (x,\bar{x};z,\bar{z})]. \tag{4.15}$$

If we assume Φ_1 to be embedded into a primary of the current algebra, this equation is true up to first order in the fermionic currents as demonstrated in some detail in appendix B. Thus, from now on we work at leading order in the fermionic currents. Similarly we have the equation

$$\partial_{\bar{z}} \Phi_1(x,\bar{x};z,\bar{z}) = -\frac{1}{c_-}\partial_{\bar{x}}[: j_{L,\bar{z}}\Phi_1 : (x,\bar{x};z,\bar{z})] \tag{4.16}$$

Using the expression of the stress-tensor in terms of the right-current, we obtain similar relations:

$$\begin{aligned}
\partial_z \Phi_1(x,\bar{x};z,\bar{z}) &= -\frac{1}{c_-}\partial_x[: j_{R,z}\Phi_1 : (x,\bar{x};z,\bar{z})] && (4.17)\\
\partial_{\bar{z}} \Phi_1(x,\bar{x};z,\bar{z}) &= -\frac{1}{c_+}\partial_{\bar{x}}[: j_{R,\bar{z}}\Phi_1 : (x,\bar{x};z,\bar{z})]. && (4.18)
\end{aligned}$$

From these relations and by integrating over $\bar{x}$, we deduce the following equations satisfied by the operator Λ:

$$\begin{aligned}
\partial_z \Lambda(x,\bar{x};z,\bar{z}) &= -\frac{\pi}{c_-} : j_{R,z}\Phi_1 : (x,\bar{x};z,\bar{z})\\
\partial_{\bar{z}} \Lambda(x,\bar{x};z,\bar{z}) &= -\frac{\pi}{c_+} : j_{R,\bar{z}}\Phi_1 : (x,\bar{x};z,\bar{z})\,. && (4.19)
\end{aligned}$$

All these relations will be repeatedly used in the derivation of the algebra of currents in the boundary theory.

4.2 Computation of the spacetime R-current algebra

In order to show that the operator $J^a(x)$ defined in equation (4.10) is the R-current of the spacetime CFT, we want to compute the spacetime operator product singularity inside correlation functions in the following limit:

$$\lim_{x\to y} J^a(x)\cdot J^b(y)\,. \tag{4.20}$$

However it turns out to be more convenient to compute an OPE involving an anti-holomorphic derivative of one of the currents, namely

$$\lim_{x\to y} \partial_{\bar{x}} J^a(x)\cdot J^b(y)\,, \tag{4.21}$$

and to integrate the result with respect to $\bar{x}$. We will therefore compute the OPE between the current

$$J^b(y) = -\frac{1}{\pi}\int d^2w \left[j^b_{L,\bar{z}}\partial_w \Lambda(y,\bar{y};w,\bar{w}) + j^b_{L,z}\partial_{\bar{w}}\Lambda(y,\bar{y};w,\bar{w})\right]\,, \tag{4.22}$$

and its derivative with respect to $\bar{x}$, which, using equation (4.11), can be written as

$$\partial_{\bar{x}} J^a(x) = -\int d^2z \left[j^a_{L,\bar{z}} \partial_z \Phi_1(x,\bar{x};z,\bar{z}) + j^a_{L,z} \partial_{\bar{z}} \Phi_1(x,\bar{x};z,\bar{z}) \right] \tag{4.23}$$

where a and b are SU(2) indices. Following [10], we regularize by cutting small holes in the worldsheet, at the points where operators are inserted. This implies that we can freely use the equation of motion (that have contact terms singularities with the other operators on the worldsheet), but the integration by parts gives rise to boundary terms.[5] We can think of the operators $J^b(y)$ and $\partial_{\bar{x}} J^a(x)$ as being inserted within a worldsheet correlation function. Since we are interested in the (spacetime) OPE between these two operators, we will only keep track of the terms arising when these operators are close one to another on the worldsheet, and discard the possible contribution due to the presence of other operators in the correlation function. A general justification for this procedure can be found in [24]. So we will write the $\bar{x}-$derivative of the R-current as

$$\partial_{\bar{x}} J^a(x) = \frac{1}{i} \left[\oint_w d\bar{z} j^a_{L,\bar{z}} \Phi_1(x,\bar{x};z,\bar{z}) + \oint_w dz j^a_{L,z} \Phi_1(x,\bar{x};z,\bar{z}) \right], \tag{4.24}$$

where the contour integral runs over the boundary of the small disc cut out around the position of the integrated operator in $J^b(y)$. The OPE we wish to compute is

$$\begin{aligned} \partial_{\bar{x}} J^a(x) \cdot J^b(y) = -\frac{1}{\pi i} \int d^2w \Big[& \left(j^b_{L,\bar{z}} \partial_w \Lambda(y,\bar{y};w,\bar{w}) + j^b_{L,z} \partial_{\bar{w}} \Lambda(y,\bar{y};w,\bar{w}) \right) \\ & \cdot \left(\oint_w d\bar{z} j^a_{L,\bar{z}} \Phi_1 + \oint_w dz j^a_{L,z} \Phi_1 \right) \Big] \end{aligned} \tag{4.25}$$

The contour integrals around the point w will pick up the singular terms in the OPE between the integrated composite operators. First notice that there are no singular terms arising between the SU(2) currents and the operators Φ_1, $\partial_w \Lambda$ and $\partial_{\bar{w}} \Lambda$ since the later transform in the trivial representation under the action of SU(2). Moreover there is no singularity either in the OPE between Φ_1 and $\partial_w \Lambda$, $\partial_{\bar{w}} \Lambda$. To prove this we use the formula (4.19):

$$\begin{aligned} \lim_{z \to w} \Phi_1(x,\bar{x};z,\bar{z}) \partial_w \Lambda(y,\bar{y};w,\bar{w}) &= -\frac{\pi}{c_-} \lim_{z \to w} \Phi_1(x,\bar{x};z,\bar{z}) : j_{R,z} \Phi_1 : (y,\bar{y};w,\bar{w}) \\ &= -\frac{\pi}{c_-} \frac{c_-}{c_+ + c_-} \frac{\partial_{\bar{x}} [(\bar{x}-\bar{y})^2 \delta^{(2)}(x-y) \Phi_1(y,\bar{y};w,\bar{w})]}{z-w} \\ &\quad -\frac{\pi}{c_-} \delta^{(2)}(x-y) : j_{R,z} \Phi_1 : (y,\bar{y};w,\bar{w}) + \mathcal{O}(z-w,\bar{z}-\bar{w}) \\ &= \delta^{(2)}(x-y) \partial_w \Lambda(y,\bar{y};w,\bar{w}) + \mathcal{O}(z-w,\bar{z}-\bar{w}). \end{aligned} \tag{4.26}$$

The previous computation is straightforwardly generalized to the OPE between the operators Φ_1 and $\partial_{\bar{w}} \Lambda$. We conclude that the only singular terms picked up by the contour

[5]We can also work with a worldsheet without holes. In that case we can integrate by parts freely, but the contact terms between the equations of motion and the other operators will contribute.

integral in equation (4.25) come from the OPE between two SU(2) currents. We obtain:

$$\begin{aligned}
\partial_{\bar{x}} J^a_R(x) J^b_R(y) = & -\frac{1}{2\pi i}\int d^2w \\
& \times\Bigg\{ \oint_w d\bar{z}\left(\frac{\kappa^{ab}c_3}{(\bar{z}-\bar{w})^2}+f^{ab}{}_c c_4\frac{j^c_{\bar{z}}(w)}{\bar{z}-\bar{w}}+f^{ab}{}_c(c_4-g)\frac{j^c_z(w)(z-w)}{(\bar{z}-\bar{w})^2}\right) \\
& \quad\times\left(\lim_{:z\to w:}\Phi_1(z,\bar{z};x,\bar{x})\partial_w\Lambda(w,\bar{w};y,\bar{y})\right) \\
& \quad+\oint_w d\bar{z}\left(\tilde{c}\kappa^{ab}2\pi\delta^{(2)}(z-w)+f^{ab}{}_c\frac{(c_4-g)j^c_z(w)}{\bar{z}-\bar{w}}+f^{ab}{}_c\frac{(c_2-g)j^c_{\bar{z}}(w)}{z-w}\right) \\
& \quad\times\left(\lim_{:z\to w:}\Phi_1(z,\bar{z};x,\bar{x})\partial_w\Lambda(w,\bar{w};y,\bar{y})\right) \\
& \quad+\oint_w dz\left(\tilde{c}\kappa^{ab}2\pi\delta^{(2)}(z-w)+f^{ab}{}_c\frac{(c_4-g)j^c_z(w)}{\bar{z}-\bar{w}}+f^{ab}{}_c\frac{(c_2-g)j^c_{\bar{z}}(w)}{z-w}\right) \\
& \quad\times\left(\lim_{:z\to w:}\Phi_1(z,\bar{z};x,\bar{x})\partial_{\bar{w}}\Lambda(w,\bar{w};y,\bar{y})\right) \\
& \quad+\oint_w dz\left(\frac{\kappa^{ab}c_1}{(z-w)^2}+f^{ab}{}_c c_2\frac{j^c_z(w)}{z-w}+f^{ab}{}_c(c_2-g)\frac{j^c_{\bar{z}}(w)(\bar{z}-\bar{w})}{(z-w)^2}\right) \\
& \quad\times\left(\lim_{:z\to w:}\Phi_1(z,\bar{z};x,\bar{x})\partial_{\bar{w}}\Lambda(w,\bar{w};y,\bar{y})\right)\Bigg\}.
\end{aligned} \tag{4.27}$$

We get twelve terms (three on each double-line) that we denote $A_1,\dots,A_{12}$. We can now explicitly perform the contour integrals. One observes that the terms A_3, A_6, A_8 and A_{12} vanish. The contour integrals in A_2, A_9, A_5 and A_{11} can be simply performed. The regular limit $\lim_{:z\to w:}\Phi_1(z,\bar{z};x,\bar{x})\partial_w\Lambda(w,\bar{w};y,\bar{y})$ can then be read from equation (4.26) (and similarly for the limit involving the anti-holomorphic derivative of Λ). Using the relation $c_2+c_4-g=i$ from equations (3.6), the terms (A_2+A_9) and (A_5+A_{11}) can be simplified separately and these four terms combine to give

$$A_2+A_9+A_5+A_{11}=2\pi\delta^{(2)}(x-y)\,if^{ab}{}_c J^c(y)\,. \tag{4.28}$$

This leaves us with computing the terms A_1, A_4, A_7, A_{10}, which involve double poles. We deal with A_1 explicitly first:

$$\begin{aligned}
A_1 &= -\frac{1}{\pi i}\int d^2w\oint_w d\bar{z}\frac{\kappa^{ab}c_3}{(\bar{z}-\bar{w})^2}\left[\lim_{:z\to w:}\Phi_1(x,\bar{x};z,\bar{z})\partial_w\Lambda(y,\bar{y};w,\bar{w})\right] \\
&= -2\kappa^{ab}c_3\int d^2w\left[\lim_{:z\to w:}\partial_{\bar{z}}\Phi_1(x,\bar{x};z,\bar{z})\partial_w\Lambda(y,\bar{y};w,\bar{w})\right] \\
&= \frac{2\kappa^{ab}c_3}{c_-}\int d^2w\partial_x\left[\lim_{:z\to w:}:j_{L,\bar{z}}\Phi_1:(x,\bar{x};z,\bar{z})\partial_w\Lambda(y,\bar{y};w,\bar{w})\right]\,.
\end{aligned} \tag{4.29}$$

where we performed the contour integral and replaced the derivative of Φ_1 using the formula (4.16) We have to compute the regular term in the OPE between $j_{L,\bar{z}}\Phi_1:(z,\bar{z};x,\bar{x})$

and $\partial_w\Lambda(w,\bar{w};y,\bar{y})$. Using formula (4.19), we have:

$$\begin{aligned}
&\lim_{:z\to w:} :j_{L,\bar{z}}\Phi_1:(x,\bar{x};z,\bar{z})\partial_w\Lambda(y,\bar{y};w,\bar{w}) \\
&\quad= -\frac{\pi}{c_-}\lim_{:z\to w:} :j_{L,\bar{z}}\Phi_1:(x,\bar{x};z,\bar{z}):j_{R,z}\Phi_1:(y,\bar{y};w,\bar{w}) \\
&\quad= -\frac{\pi}{c_-}\delta^{(2)}(x-y):j_{L,\bar{z}}:j_{R,z}\Phi_1::(y,\bar{y};w,\bar{w}) \\
&\quad= \delta^{(2)}(x-y):j_{L,\bar{z}}\partial_w\Lambda:(y,\bar{y};w,\bar{w})
\end{aligned} \tag{4.30}$$

We deduce that

$$A_1 = 2\kappa^{ab}\frac{c_3}{c_-}\partial_x\delta^{(2)}(x-y)\int d^2w:j_{L,\bar{z}}\partial_w\Lambda:(y,\bar{y};w,\bar{w})\,. \tag{4.31}$$

A similar analysis for the double pole term in A_{10} leads to

$$A_{10} = 2\kappa^{ab}\frac{c_1}{c_+}\pi\partial_x\delta^{(2)}(x-y)\int d^2w:j_{L,z}\partial_{\bar{w}}\Lambda:(y,\bar{y};w,\bar{w}). \tag{4.32}$$

The contact terms in A_4 and A_7 contribute due to the following integrals:

$$\begin{aligned}
\oint_w dz\delta^{(2)}(z-w)f(z,\bar{z}) &= \frac{-1}{2\pi}\oint_w dz\partial_{\bar{w}}\frac{1}{z-w}f(z,\bar{z}) = -i\partial_{\bar{w}}f(w,\bar{w}) \\
\oint_w d\bar{z}\delta^{(2)}(z-w)f(z,\bar{z}) &= \frac{-1}{2\pi}\oint_w dz\partial_w\frac{1}{\bar{z}-\bar{w}}f(z,\bar{z}) = -i\partial_w f(w,\bar{w})\,.
\end{aligned} \tag{4.33}$$

Using these results and following the steps we did for the evaluation of A_1, we find that

$$\begin{aligned}
A_4 &= 2\kappa^{ab}\tilde{c}\int d^2w\left[\lim_{:z\to w:}\partial_{\bar{z}}\Phi_1(x,\bar{x};z,\bar{z})\partial_w\Lambda(y,\bar{y};w,\bar{w})\right] \\
&= -2\kappa^{ab}\frac{\tilde{c}}{c_-}\partial_x\delta^{(2)}(x-y)\int d^2w:j_{L,\bar{z}}\partial_w\Lambda:(y,\bar{y};w,\bar{w}) \\
\text{and}\quad A_7 &= -2\kappa^{ab}\frac{\tilde{c}}{c_+}\partial_x\delta^{(2)}(x-y)\int d^2w:j_{L,z}\partial_{\bar{w}}\Lambda:(y,\bar{y};w,\bar{w})\,.
\end{aligned} \tag{4.34}$$

Combining these four terms and using the relations between coefficients (3.6), we get

$$A_1 + A_4 + A_7 + A_{10} = -2\pi\kappa^{ab}\partial_x\delta^{(2)}(x-y)\,I(y,\bar{y})\,, \tag{4.35}$$

where we have introduced the central extension operator I of the R-current algebra:

$$I(y,\bar{y}) = \frac{1}{\pi}\int d^2w\left[j_{L,w}\partial_{\bar{w}}\Lambda(y,\bar{y};w,\bar{w}) + j_{L,\bar{w}}\partial_w\Lambda(y,\bar{y};w,\bar{w})\right]\,. \tag{4.36}$$

We shall turn to the study of this operator after deriving the algebra involving the other bosonic currents on the boundary. For now, we observe that when $f^2 = 1/k$, one can check that this reduces to the operator I defined in [10].[6] We can also check that the operator

[6]Our definition of I differs from the one of [10] by an overall normalization factor of k.

$I(y,\bar{y})$ does not depend on the spacetime coordinates $y,\bar{y}$:

$$\begin{aligned}
\partial_{\bar{y}} I(y,\bar{y}) &= \int d^2w\, [j_{L,w}\partial_{\bar{w}}\Phi_1(y,\bar{y};w,\bar{w}) + j_{L,\bar{w}}\partial_w\Phi_1(y,\bar{y};w,\bar{w})] \\
&= \frac{1}{i}\oint dw\, [j_{L,w}\Phi_1(y,\bar{y};w,\bar{w})] - \frac{1}{i}\oint d\bar{w}\, [j_{L,\bar{w}}\Phi_1(y,\bar{y};w,\bar{w})] \\
&= -\frac{c_+}{\pi i}\oint dw\, [\partial_w\bar{\Lambda}(y,\bar{y};w,\bar{w})] - \frac{c_-}{\pi i}\oint d\bar{w}\, [\partial_{\bar{w}}\bar{\Lambda}(y,\bar{y};w,\bar{w})] \\
&= 0\,.
\end{aligned} \tag{4.37}$$

From the first line to the second, we integrated by parts and used the current conservation. From the second to the third, we used the (spacetime) complex conjugate of equation (4.19) that involves the operator $\bar{\Lambda}$ defined as: $\partial_x\bar{\Lambda} = \Phi_1$. Similarly we can prove that $\partial_y I(y,\bar{y}) = 0$.

Putting together what we have so far, we find the OPE

$$\partial_{\bar{x}} J^a(x)\cdot J^b(y) \sim -2\pi\kappa^{ab}\partial_x\delta^{(2)}(x-y)\, I + 2\pi\delta^{(2)}(x-y)\, i f^{ab}{}_c J^a(y)\,, \tag{4.38}$$

We can integrate with respect to $\bar{x}$ and find:

$$J^a(x)\cdot J^b(y) \sim \kappa^{ab}\frac{1}{(x-y)^2}\, I + \frac{1}{x-y}\, i f^{ab}{}_c J^c(y)\,. \tag{4.39}$$

We observe that the $SU(2)_R$ symmetry of the $N=4$ superconformal algebra is at level I. By the structure of the $N=4$ superconformal algebra, this implies a central charge equal to $c = 6\,I$. We will confirm the value of the central charge by evaluating the operator product expansion of the stress-tensor with itself.

5 The spacetime Virasoro algebra

In the previous section we have identified the vertex operators that correspond, in spacetime, to the (left) R-currents $J^a(x)$. They are part of the generators of the small $N=(4,4)$ superconformal symmetry of the holographically (boundary) dual conformal field theory. In this section we will address the construction of the full set of left generators. The generators of the space-time right-moving superconformal algebra can be constructed similarly. We put forward the following expression for the vertex operator of the boundary stress tensor:

$$\begin{aligned}
T(x) = -\frac{1}{2\pi}\int d^2z\, \big[&(\partial_x j_{L,z}\partial_x\partial_{\bar{z}}\Lambda(x,\bar{x};z,\bar{z}) + 2\partial_x^2 j_{L,z}\partial_{\bar{z}}\Lambda(x,\bar{x};z,\bar{z})) \\
&+(\partial_x j_{L,\bar{z}}\partial_x\partial_z\Lambda(x,\bar{x};z,\bar{z}) + 2\partial_x^2 j_{L,\bar{z}}\partial_z\Lambda(x,\bar{x};z,\bar{z}))\big]\,.
\end{aligned} \tag{5.1}$$

Once again, this is a non-chiral generalization of the expression for the stress tensor for the case with pure NS flux [10]. First we will compute the OPE between this operator and the R-current (4.10). We shall show that, given our basic OPEs of the worldsheet currents and primary operators, this reproduces the superconformal OPEs of the boundary operators.

Before we delve into this calculation, it is important to note that the operator is closed in the hybrid cohomology. The operator is independent of the ghosts and therefore it is

BRST closed when it is a worldsheet conformal primary of conformal dimension one — this follows from formula (2.7). Using the operator product expansion between the worldsheet energy-momentum tensor, the currents and the field Λ, it is possible to show that this is true for the particular coefficients chosen in equation (5.1). One can alternatively fix the coefficients in formula (5.1) by demanding that the energy-momentum tensor transform as a weight 2 tensor under the global $SL(2,\mathbb{R})_L$ symmetry group (up to a BRST exact operator). As a BRST-closed operator, we can insert it in hybrid superstring amplitudes, and compute Ward identities for the resulting correlator.

As for the previous computation, it is simpler to consider the OPE between the $\bar{x}$-derivative of the current and the stress-energy tensor. We wish to evaluate the OPE:

$$
\begin{aligned}
\partial_{\bar{x}} J^a(x)\cdot T(y) = &-\frac{1}{2\pi i}\left[\oint_w dz j^a_{L,z}\Phi_1(x,\bar{x};z,\bar{z}) + \oint_w d\bar{z} j^a_{L,\bar{z}}\Phi_1(x,\bar{x};z,\bar{z})\right] \\
&\times\left[\int d^2w\left[(\partial_y j_{L,z}\partial_y\partial_{\bar{w}}\Lambda(y,\bar{y};w,\bar{w}) + 2\partial_y^2 j_{L,z}\partial_{\bar{w}}\Lambda(y,\bar{y};w,\bar{w}))\right.\right. \\
&\left.\left.+(\partial_y j_{L,\bar{z}}\partial_y\partial_w\Lambda(y,\bar{y};w,\bar{w}) + 2\partial_y^2 j_{L,\bar{z}}\partial_w\Lambda(y,\bar{y};w,\bar{w}))\right]\right].
\end{aligned} \tag{5.2}
$$

The contour integral will pick up the poles in the OPEs between the integrated operators. The SU(2) currents $j^a_{L,\bar{z}}$ and $j^a_{L,z}$ will not contribute to these poles since all of the integrated operators in the stress-tensor transform trivially under the $SU(2)_L$ symmetry. So the singular terms only come from the OPE between the operator Φ_1 and the integrated operators in the stress-tensor. Notice that there is no short-distance singularity between Φ_1 and $\partial_z\Lambda$. Indeed using equation (4.19) we can rewrite the later as $: j_{R,z}\Phi_1 :$. The right current has a pole in its OPE with Φ_1, but the coefficient cancels against the delta-function appearing when the two Φ_1 operators come close to each other. The same is true for $\partial_{\bar{z}}\Lambda$. We conclude that the only singular terms that will contribute to the OPE (5.2) come from the short-distance singularity between the Φ_1 operators integrated in the derived R-current, and the $SL(2,\mathbb{R})$-left currents integrated in the stress-tensor.

We can expand the OPE (5.2) as a sum of eight terms. Consider, for instance, the term obtained by taking the OPE between the first terms in each of the bracketed terms in (5.2). As argued before, the contour integral picks up the pole coming from the short distance singularity between the operator Φ_1 and the $SL(2,\mathbb{R})$ current:

$$
\begin{aligned}
B_1 &= -\frac{1}{2\pi i}\int d^2w\left[\oint_w dz j^a_{L,\bar{z}}\Phi_1(x,\bar{x};z,\bar{z})\partial_y j_{L,z}\partial_y\partial_w\Lambda(y,\bar{y};w,\bar{w})\right] \\
&= -\frac{1}{2\pi i}\int d^2w\left[\oint_w dz j^a_{L,\bar{z}}\lim_{:z\to w:}\partial_y\left(\frac{\partial_x[(x-y)^2\Phi_1(x,\bar{x};z,\bar{z})]}{w-z}\right)\partial_y\partial_w\Lambda(y,\bar{y};w,\bar{w})\right] \\
&= \int d^2w\left[j^a_{L,\bar{z}}\lim_{:z\to w:}\partial_x[2(y-x)\Phi_1(x,\bar{x};z,\bar{z})\partial_y\partial_w\Lambda(y,\bar{y};w,\bar{w})]\right] \\
&= -2\partial_x\delta^{(2)}(x-y)\int d^2w\left[j^a_{L,\bar{z}}\partial_w\Lambda(y,\bar{y};w,\bar{w})\right].
\end{aligned} \tag{5.3}
$$

Using the same techniques, we can compute all of the remaining terms. We suppress the details since the calculations are very similar to the ones we have already performed.

Combining these terms, we eventually obtain the spacetime OPE

$$\partial_{\bar{x}} J^a(x)T(y) \sim -2\pi \partial_x \delta^{(2)}(x-y)J^a(y)\,, \tag{5.4}$$

which can be integrated with respect to $\bar{x}$ to give:

$$J^a(x)T(y) \sim \frac{J^a(y)}{(x-y)^2}. \tag{5.5}$$

By evaluating the right-hand side at the point x, we get that the current is a conformal primary of dimension one in space-time:

$$T(y)J^a(x) \sim \frac{J^a(x)}{(x-y)^2} + \frac{\partial_x J^a(x)}{y-x}. \tag{5.6}$$

One can also check that with our definition, the stress-tensor satisfies the OPE that codes the Virasoro algebra:

$$T(x)\cdot T(y) \sim \frac{3I}{(x-y)^4} + \frac{2T(y)}{(x-y)^2} + \frac{\partial T(y)}{(x-y)}\,, \tag{5.7}$$

which gives for the central charge $c = 6\,I$, where I is the central extension. The details are given in the appendix C and confirms the value of the central charge obtained using the R-current algebra. This is a consistency check on our proposal for the vertex operators of the boundary currents. In the following, we will discuss some of the properties of the central extension in more detail. But first, we briefly outline how our discussion may be extended to also describe the superconformal generators.

5.1 Superconformal generators

With respect to the scaling operator, we have the $sl(2,r)$ generators of charges $\pm 1, 0$, and the $su(2)$ charges which are inert. The eight fermionic generators have charges $\pm\frac{1}{2}$. It is natural then to gather the eight fermionic currents into four x-dependent currents:

$$\begin{aligned} j_F^{\tilde{a}\alpha}(x) &= e^{-x\mathcal{J}_0^-}\, j_F^{+\frac{1}{2},\tilde{a}\alpha} e^{+x\mathcal{J}_0^-} \\ &= j_F^{+\frac{1}{2},\tilde{a}\alpha} - x j_F^{-\frac{1}{2},\tilde{a}\alpha}, \end{aligned} \tag{5.8}$$

where $\tilde{a}$ indicates an $su(2)_R$ doublet. The current must have weight $(-\frac{1}{2}, 0)$ in space-time, therefore, the superconformal generators should be of the general form

$$G^{\tilde{a}\alpha}(x) = \int_\Sigma b_1 j_F^{\tilde{a}\alpha}(x) \otimes j_R \partial_x \Phi_1 + b_2 \partial_x j_F^{\tilde{a}\alpha}(x) \otimes j_R \Phi_1. \tag{5.9}$$

These operators have weight $(\frac{3}{2}, 0)$ in space-time. Imposing that the operator transforms appropriately with respect to the raising operator in the $sl(2,\mathbb{R})$ algebra gives rise to the relation

$$b_2 = 2b_1. \tag{5.10}$$

We thus propose the following operator to generate the supersymmetry transformations of the boundary theory:

$$G^{\tilde{a}\alpha}(x) = b \int_\Sigma j_F^{\tilde{a}\alpha}(x) \otimes j_R \partial_x \Phi_1 + 2\partial_x j_F^{\tilde{a}\alpha}(x) \otimes j_R \Phi_1. \tag{5.11}$$

This is the analogue of the expressions in [10] in the zero ghost picture.

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Higher superfield components. In order to prove that we have an $N=4$ superconformal algebra in space-time, we should proceed along the lines of the calculation of the R-current algebra and compute the OPE of $\partial_{\bar{x}} G^{\tilde{a}\alpha}(x) \cdot G^{\tilde{b}\beta}(y)$ within a correlation function. There are some qualitatively new features that arise when we attempt to compute it, such as the OPE between the fermionic currents and the operator Φ_1.

This calculation can guide us in lifting the limitations on our derivation of the R-current and Virasoro algebras. The vertex operators for the physical fields described in [11] have a full supermultiplet worth of fields and should be thought of as superfields in spacetime (see e.g. [15, 16]). However, as is clear from our expressions of the vertex operators in equations (4.10) and (5.1), we have chosen to truncate to a bosonic component of the superfield in spacetime. To compute the super conformal algebra in space-time we need to take into account the first fermionic correction to these superfields, and we need to work at least to first order in the fermionic currents.

We leave this for future work and proceed to discuss properties of the central extension operator.

5.2 Further remarks

From the operator product expansion involving the boundary stress tensor and the R-currents, we found that the central charge of the boundary theory is given by the operator $6\,I$, where I is the operator

$$I = \frac{1}{\pi} \int d^2 w \, [j_{L,w}(w;x) \partial_{\bar{w}} \Lambda(w;x) + j_{L,\bar{w}}(w;x) \partial_w \Lambda(w;x)] \ . \tag{5.12}$$

We would like to make a few observations regarding the central extension operator.

1. The operator $I(x,\bar{x})$ is independent of the spacetime coordinates $(x,\bar{x})$. The proof for this has been given in equation (4.37). Thus it behaves like a constant in correlation functions.

2. Near the boundary of AdS space, in the $\phi \to \infty$ limit, the operator I takes the form

$$I = -\int d^2 z (c_+ \bar{\partial}\gamma \partial\bar{\gamma} + c_- \partial\gamma \bar{\partial}\bar{\gamma}) \delta^{(2)}(x-\gamma) - \frac{1}{\pi} \int d^2 z e^{2\phi} (c_+ \partial\bar{\gamma} \bar{\partial}\gamma + c_- \partial\gamma \bar{\partial}\bar{\gamma}) + \dots \tag{5.13}$$

   From the form of the vertex operator we can see that it is an off-diagonal mode of the metric (mixed with an anti-symmetric tensor).

3. For the case of pure NSNS flux, the operator I that we found agrees with the central extension operator found in [10].[7] We refer the reader to [13] for a detailed analysis regarding the behaviour and the interpretation of the operator I in correlation functions.

Note in particular that by the fact that the operator $6\,I$ appears as the coefficient of the boundary two-point function for the boundary energy-momentum tensor, we know

[7] Our definition differs from the one in [10] by an overall normalization factor of $1/k$.

that it governs the value of the conformal anomaly. In a background AdS_3 space-time, there are various other ways to compute the value of the conformal anomaly coefficient in the supergravity approximation, namely from the Virasoro algebra directly [7] or from the holographic Weyl anomaly [27]. These imply that in an AdS_3 background the vacuum expectation value of the central extension operator is equal to the Brown-Henneaux central charge as determined in supergravity. This has been argued in more detail in [9, 10, 13] for the NS-NS case. Our calculation is also valid in the strongly curved regime, and codes corrections to the central extension operator that can occur in excited states (as in [10, 13]).

6 Summary and conclusions

Our main result is the construction of the vertex operators for the boundary superconformal algebra, in the worldsheet theory that describes string theory on $AdS_3 \times S^3$ with both Ramond-Ramond and Neveu-Schwarz-Neveu-Schwarz fluxes. We have verified that they satisfy the conformal operator algebra using stringy worldsheet techniques. A crucial technical ingredient was the conformal current algebra [12] satisfied by the currents in the supergroup model $PSU(1,1|2)$. It is important to note that most of the simplifications that arise in the calculations follow from constraints on the coefficients appearing in the current algebra. These constraints, in turn, follow from the existence of a supergroup valued field throughout the two-dimensional moduli space parameterized by the fluxes.

It is a good check on our construction that at Wess-Zumino-Witten points (where the coefficient of the kinetic term is equal in absolute value to the coefficient of the Wess-Zumino term), which is the case with only NS-NS flux, our results coincide with those obtained in the NSR formalism in [9, 10, 13]. Technically, our results are a non-chiral generalization of those references. Our generalization includes backgrounds which are near-brane limits of D1-D5 D-brane systems for which the infrared limit of the dual gauge theory in the Higgs phase can be studied directly. An interesting result is the explicit expression for the central extension operator.

There are many directions for future work. A clear challenge is to extend our analysis to the full $N = 4$ superconformal algebra. A related problem is to further exploit the $PSU(1,1|2)$ superisometries of the hybrid formalism to covariantize further our analysis. One can also attempt to generalize our arguments to other AdS_3 backgrounds that can be described in a Berkovits type formalism.

We would like to argue that our construction is a significant first step in the construction of the full spectrum of string theory (including black hole excitations) in the AdS_3 background with Ramond-Ramond fluxes, with appropriate boundary conditions. Indeed, the full local asymptotic symmetry algebra in space-time will be the natural tool to classify the spectrum, thus reducing the spectral problem to listing primary states. The latter often reduces to a mini-superspace problem.

Our construction of the spacetime symmetry generators may be extended to $AdS_2 \times S^2$ backgrounds. String theory in $AdS_2 \times S^2$ with Ramond-Ramond fluxes can be described using the four-dimensional hybrid formalism [28]. The target space in that case is the geometric coset $PSU(1,1|2)/(\mathrm{U}(1) \times \mathrm{U}(1))$ [29]. Since the gauging involves only the right symmetry, a reasonable hypothesis is that the left-current algebra will be left intact. Then our previous construction will extend straightforwardly. Notice that we obtain a set of holomorphic generators in spacetime, as suited for a chiral two-dimensional CFT dual to quantum gravity on an AdS_2 spacetime.

Finally, the conformal current algebra of the supergroup sigma-model is tied in with both the worldsheet integrable structure of the sigma-model, and the infinite set of conserved charges in space-time. It would be useful to understand better the relations between these two important features of the theory.

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A Worldsheet action for strings on AdS_3 with RR flux

A.1 Matrix generators for $PSU(1,1|2)$

We will now give an explicit matrix representation of the superalgebra which we will use to write down the worldsheet action. Recall that the Pauli sigma-matrices are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.1}$$

They satisfy the algebra

$$\sigma_a \sigma_b = \delta_{ab} I + i\epsilon_{abc}\sigma_c \,. \tag{A.2}$$

Then we can write the bosonic generators K_{ab} in terms of the Pauli matrices:

$$K_{12} = \begin{pmatrix} -\frac{i}{2}\sigma_3 & 0 \\ 0 & -\frac{i}{2}\sigma_3 \end{pmatrix} \quad K_{13} = \begin{pmatrix} \frac{i}{2}\sigma_2 & 0 \\ 0 & \frac{i}{2}\sigma_2 \end{pmatrix} \quad K_{23} = \begin{pmatrix} -\frac{i}{2}\sigma_1 & 0 \\ 0 & -\frac{i}{2}\sigma_1 \end{pmatrix} \tag{A.3}$$

$$K_{14} = \begin{pmatrix} -\frac{i}{2}\sigma_1 & 0 \\ 0 & \frac{i}{2}\sigma_1 \end{pmatrix} \quad K_{24} = \begin{pmatrix} -\frac{i}{2}\sigma_2 & 0 \\ 0 & \frac{i}{2}\sigma_2 \end{pmatrix} \quad K_{34} = \begin{pmatrix} -\frac{i}{2}\sigma_3 & 0 \\ 0 & \frac{i}{2}\sigma_3 \end{pmatrix}. \tag{A.4}$$

Similarly, we represent the fermionic generators as

$$\begin{aligned}
S_{11} &= \begin{pmatrix} 0 & \frac{1}{2}\sigma_1 \\ -\frac{1}{2}\sigma_1 & 0 \end{pmatrix} & S_{21} &= \begin{pmatrix} 0 & \frac{1}{2}\sigma_2 \\ -\frac{1}{2}\sigma_2 & 0 \end{pmatrix} \\
S_{31} &= \begin{pmatrix} 0 & \frac{1}{2}\sigma_3 \\ -\frac{1}{2}\sigma_3 & 0 \end{pmatrix} & S_{41} &= \begin{pmatrix} 0 & -\frac{i}{2}\mathbb{I} \\ -\frac{i}{2}\mathbb{I} & 0 \end{pmatrix} \\
S_{12} &= \begin{pmatrix} 0 & -\frac{1}{2}\sigma_1 \\ -\frac{1}{2}\sigma_1 & 0 \end{pmatrix} & S_{22} &= \begin{pmatrix} 0 & -\frac{1}{2}\sigma_2 \\ -\frac{1}{2}\sigma_2 & 0 \end{pmatrix} \\
S_{32} &= \begin{pmatrix} 0 & -\frac{1}{2}\sigma_3 \\ -\frac{1}{2}\sigma_3 & 0 \end{pmatrix} & S_{42} &= \begin{pmatrix} 0 & \frac{i}{2}\mathbb{I} \\ -\frac{i}{2}\mathbb{I} & 0 \end{pmatrix}.
\end{aligned}$$

They give the generators in the fundamental representation of $sl(2|2)$. The generators S square to a multiple of the identity. We choose the invariant metric:

$$\langle K_{ab}, K_{cd}\rangle = -\epsilon_{abcd} = Str(K_{ab}K_{cd}) \qquad \langle S_{a\alpha}, S_{b\beta}\rangle = -\epsilon_{\alpha\beta}\delta_{ab} = Str(S_{a\alpha}S_{b\beta}) \tag{A.5}$$

It will turn out to be useful to choose the following basis of bosonic generators of $psu(1,1|2)$:

$$K_0 = \begin{pmatrix} -\frac{i}{2}\sigma_3 & 0 \\ 0 & 0 \end{pmatrix} \quad K_1 = \begin{pmatrix} \frac{1}{2}\sigma_2 & 0 \\ 0 & 0 \end{pmatrix} \quad K_2 = \begin{pmatrix} -\frac{1}{2}\sigma_1 & 0 \\ 0 & 0 \end{pmatrix} \tag{A.6}$$

$$K_3 = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{i}{2}\sigma_3 \end{pmatrix} \quad K_4 = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{i}{2}\sigma_1 \end{pmatrix} \quad K_5 = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{i}{2}\sigma_2 \end{pmatrix} \tag{A.7}$$

These are related to the K_{ab} by a simple linear change of basis:

$$\begin{aligned} K_0 &= \frac{1}{2}(K_{12}+K_{34}) \qquad K_1 = \frac{1}{2i}(K_{13}-K_{24}) \qquad K_2 = \frac{1}{2i}(K_{14}+K_{23}) \\ K_3 &= \frac{1}{2}(K_{12}-K_{34}) \qquad K_4 = \frac{1}{2}(K_{23}-K_{14}) \qquad K_5 = -\frac{1}{2}(K_{13}+K_{24})\,. \end{aligned} \tag{A.8}$$

These generators satisfy

$$\langle K_i, K_j\rangle = \frac{1}{2}\eta_{ij} = Str(K_iK_j) \tag{A.9}$$

with signature $(-+++++)$. For the fermionic generators, we use the linear combinations

$$S^{\pm}_{1\alpha} = S_{1\alpha} \pm iS_{2\alpha} \qquad S^{\pm}_{3\alpha} = S_3 \pm iS_4\,. \tag{A.10}$$

One can easily obtain the commutation relations between these generators from the ones in (2.1). These will prove to be useful in writing out the action explicitly.

A.2 Explicit form of the worldsheet currents

Our goal in this section will be to obtain a concrete parameterization of the sigma-model on the supergroup. We parameterize the group element $g \in \mathrm{SU}(1,1|2)$ as

$$g = e^{\alpha} g_F g_{S^3} g_{AdS_3} \tag{A.11}$$

with

$$g_F = e^{\theta^{a\alpha}S_{a\alpha}} \tag{A.12}$$

$$\begin{aligned} g_{S^3} &= e^{-(\varphi_1+\varphi_2)K_3} e^{-2\theta K_4} e^{-(\varphi_1-\varphi_2)K_3} \\ &= \begin{pmatrix} \mathbb{I} & 0 \\ 0 & \begin{matrix} e^{i\varphi_1}\cos\theta & ie^{i\varphi_2}\sin\theta \\ ie^{-i\varphi_2}\sin\theta & e^{-i\varphi_1}\cos\theta \end{matrix} \end{pmatrix} \end{aligned} \tag{A.13}$$

and α is an overall phase that we will gauge away eventually. For AdS_3 we choose the Poincaré parameterization:

$$g_{AdS_3} = \frac{1}{2}\begin{pmatrix} \begin{matrix} e^{-\phi}+(\gamma-i)(\bar\gamma+i)e^{\phi} & e^{-\phi}+e^{\phi}(\gamma-i)(\bar\gamma-i) \\ e^{-\phi}+e^{\phi}(\gamma+i)(\bar\gamma+i) & e^{-\phi}+(\gamma+i)(\bar\gamma-i)e^{\phi} \end{matrix} & 0 \\ 0 & \mathbb{I} \end{pmatrix}. \tag{A.14}$$

This follows from the following parameterization of SL$(2,\mathbb{R})$:

$$g_{\mathrm{SL}(2,\mathbb{R})} = \begin{pmatrix} e^{-\phi} + \gamma\bar{\gamma}e^{\phi} & e^{\phi}\gamma & 0 \\ e^{\phi}\bar{\gamma} & e^{\phi} & \\ 0 & & \mathbb{I} \end{pmatrix}. \tag{A.15}$$

For Lorentzian AdS_3 both γ and $\bar{\gamma}$ are real while they become complex conjugate in Euclidean AdS_3. We have used the following isomorphism from SL$(2,\mathbb{R})$ to SU$(1,1)$:

$$g \to cgc^{\dagger}, \tag{A.16}$$

where the matrix c is given by:

$$c = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}. \tag{A.17}$$

We now expand the left-invariant one-form $g^{-1}dg$ on our basis of generators. We have:

$$g^{-1}dg = (g_{S^3}g_{AdS_3})^{-1}e^{-\theta^{a\alpha}F_{a\alpha}}d(e^{\theta^{a\alpha}F_{a\alpha}})g_{S^3}g_{AdS_3} + g_{S^3}^{-1}dg_{S^3} + g_{AdS_3}^{-1}dg_{AdS_3} + d\alpha\mathbb{I} \tag{A.18}$$

The S^3 part is given by

$$\begin{aligned} g_{S^3}^{-1}dg_{S^3} &= 2[\sin^2\theta d\varphi_2 - \cos^2\theta d\varphi_1]K_3 \\ &\quad +2[-\cos(\varphi_1-\varphi_2)d\theta - \cos\theta\sin\theta\sin(\varphi_1-\varphi_2)(d\varphi_1+d\varphi_2)]K_4 \\ &\quad +2[-\sin(\varphi_1-\varphi_2)d\theta + \cos\theta\sin\theta\cos(\varphi_1-\varphi_2)(d\varphi_1+d\varphi_2)]K_5 \\ &= \sum_{i=3,4,5} j^i K_i\,. \end{aligned} \tag{A.19}$$

One can check that:

$$\langle g_{S^3}^{-1}\partial g_{S^3}, g_{S^3}^{-1}\bar{\partial}g_{S^3}\rangle = 2(\partial\theta\bar{\partial}\theta + \cos^2\theta\partial\varphi_1\bar{\partial}\varphi_1 + \sin^2\theta\partial\varphi_2\bar{\partial}\varphi_2)\,. \tag{A.20}$$

So we recover the worldsheet action for a sigma-model with target space S^3. The other purely bosonic part is given by

$$\begin{aligned} g_{AdS_3}^{-1}dg_{AdS_3} &= [d\bar{\gamma} + 2\bar{\gamma}d\phi - e^{2\phi}(1+\bar{\gamma}^2)d\gamma]K_0 \\ &\quad +2[d\phi - e^{2\phi}\bar{\gamma}d\gamma]K_1 \\ &\quad +[-d\bar{\gamma} - 2\bar{\gamma}d\phi + e^{2\phi}(-1+\bar{\gamma}^2)d\gamma]K_2 \\ &= \sum_{i=0,1,2} j^i K_i\,. \end{aligned} \tag{A.21}$$

The action for the AdS part thus takes the form

$$\langle g_{AdS_3}^{-1}\partial g_{AdS_3}, g_{AdS_3}^{-1}\bar{\partial}g_{AdS_3}\rangle = e^{2\phi}\left[\bar{\partial}\bar{\gamma}\partial\gamma + \bar{\partial}\gamma\partial\bar{\gamma}\right] + 2\partial\phi\bar{\partial}\phi\,. \tag{A.22}$$

The remaining term can be tackled as follows. First, let us consider

$$\begin{aligned} g_F^{-1}dg_F &= d\theta^{a\alpha}S_{a\alpha} + \frac{1}{2}d\theta^{a\alpha}\theta^{b\beta}\epsilon_{\alpha\beta}\epsilon_{abcd}K_{cd} + \frac{1}{2}d\theta^{a\alpha}\theta^{b\beta}\theta^{c\gamma}\epsilon_{\alpha\beta}\epsilon_{abcd}S_{d\gamma} \\ &\quad +\frac{1}{6}d\theta^{a\alpha}\theta^{b\beta}\theta^{c\gamma}\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}\left[K_{ac}\theta^{b\delta} + K_{cb}\theta^{a\delta} + K_{ba}\theta^{c\delta}\right] + \ldots\,. \end{aligned} \tag{A.23}$$

Let us write this compactly as follows:

$$g_F^{-1} dg_F = f^{a\alpha} S_{a\alpha} + b^i K_i \tag{A.24}$$

In order to obtain the left-invariant one-forms, it is necessary to obtain the result of conjugating the generators by the bosonic group elements. An economical way to express the conjugation relations is as follows:

$$\begin{aligned} e^{xK_{ab}} S_{c\alpha} e^{-xK_{ab}} &= (\cos x S_{a\alpha} + \sin x S_{b\alpha}) \quad \text{for} \quad a = c \quad \text{and} \quad b \neq c \\ e^{xK_{ab}} K_{cd} e^{-xK_{ab}} &= (\cos x K_{ad} + \sin x K_{bd}) \quad \text{for} \quad a = c \quad \text{and} \quad b \neq d\,. \end{aligned} \tag{A.25}$$

Recalling that $K_{ab} = -K_{ba}$, we can obtain all other possibilities easily. This proves sufficient to compute the bosonic and fermionic currents.

Fermionic currents. Let us first compute the fermionic currents

$$\Pi_{a\alpha} = i \left\langle g^{-1} dg, S_{a\alpha} \right\rangle . \tag{A.26}$$

For this we have to compute

$$g_B^{-1} S^{\pm}_{a\alpha} g_B = (g_{\text{AdS}})^{-1} \left[(g_{S^3})^{-1} S^{\pm}_{a\alpha} \, g_{S^3} \right] g_{\text{AdS}} \,. \tag{A.27}$$

Then,

$$\Pi_{a\alpha} = \epsilon_{\alpha\beta} f^{b\beta} \delta_{ab} \,. \tag{A.28}$$

In this calculation, we find it convenient to use the global (t, φ, ρ) parameterization of AdS_3:

$$g_{AdS_3} = e^{-(t+\varphi)K_0} \, e^{-2\rho K_2} \, e^{-(t-\varphi)K_0} \,. \tag{A.29}$$

Under conjugation by the bosonic group element, the transformation of the fermionic generators is encoded in a matrix $\mathcal{M}$ such that

$$\begin{pmatrix} S^+_{1\alpha} \\ S^-_{1\alpha} \\ S^+_{3\alpha} \\ S^-_{3\alpha} \end{pmatrix} \longrightarrow \mathcal{M} \begin{pmatrix} S^+_{1\alpha} \\ S^-_{1\alpha} \\ S^+_{3\alpha} \\ S^-_{3\alpha} \end{pmatrix} . \tag{A.30}$$

where the matrix $\mathcal{M}$ is given by

$$\begin{pmatrix} e^{-i(t+\varphi_1)} c_\theta \cosh\rho & -ie^{-i(\varphi+\varphi_2)} s_\theta \sinh\rho & ie^{-i(t+\varphi_2)} \cosh\rho s_\theta & e^{-i(\varphi+\varphi_1)} c_\theta \sinh\rho \\ -ie^{i(\varphi+\varphi_2)} s_\theta \sinh\rho & e^{i(t+\varphi_1)} c_\theta \cosh\rho & -e^{i(\varphi+\varphi_1)} c_\theta \sinh\rho & -ie^{i(t+\varphi_2)} \cosh\rho s_\theta \\ ie^{-i(t-\varphi_2)} \cosh\rho s_\theta & -e^{-i(\varphi-\varphi_1)} c_\theta \sinh\rho & e^{-i(t-\varphi_1)} c_\theta \cosh\rho & ie^{-i(\varphi-\varphi_2)} s_\theta \sinh\rho \\ e^{i(\varphi-\varphi_1)} c_\theta \sinh\rho & -ie^{i(t-\varphi_2)} \cosh\rho s_\theta & ie^{i(\varphi-\varphi_2)} s_\theta \sinh\rho & e^{i(t-\varphi_1)} c_\theta \cosh\rho \end{pmatrix} . \tag{A.31}$$

We have denoted $s_\theta = \sin\theta$ and $c_\theta = \cos\theta$. The fermionic currents can then be written in the form

$$\Pi^{\pm}_{a\alpha} = f^{a\beta\pm} \mathcal{M}_{ab} \epsilon_{\beta\alpha} \,. \tag{A.32}$$

Bosonic currents. It is much easier to directly work with the K_i generators than with the K_{ab} generators to derive the contributions to the bosonic currents from the fermionic (θ-dependent) terms in the action. Starting from (A.24) and using the fact that $\{K_{0,1,2}\}$ commutes with $\{K_{3,4,5}\}$, we obtain this bosonic contribution to be

$$g^{-1}_{AdS_3}\left[\sum_{i=0,1,2} b^i K_i\right] g_{AdS_3} + g^{-1}_{S_3}\left[\sum_{i=3,4,5} b^i K_i\right] g_{S_3} = \sum_i \ell^i K_i\,. \tag{A.33}$$

Using the commutation relations, we find that

$$\begin{aligned}
\ell^0 &= \cosh 2\rho\, b_0 \sinh 2\rho \cos(t+\phi)\ b_1 - \sin(t+\phi)\sinh 2\rho\ b_2\\
\ell^1 &= \sinh 2\rho\cos(t-\phi)\ b_0 + (\cosh 2\rho\cos(t+\phi)\cos(t-\phi) - \sin(t+\phi)\sin(t-\phi))\ b_1\\
&\quad -(\cos(t+\phi)\sin(t-\phi) + \sinh 2\rho \sin(t+\phi)\cos(t-\phi))\ b_2\\
\ell^2 &= \sinh 2\rho\sin(t-\phi)\ b_0 + (\sin(t+\phi)\cos(t-\phi) + \cosh 2\rho\cos(t+\phi)\sin(t-\phi))\ b_1\\
&\quad +(\cos(t+\phi)\cos(t-\phi) - \cosh 2\rho\sin(t+\phi)\sin(t-\phi))\ b_2\\
\ell^3 &= \cos 2\theta\, b_3 + \sin(\varphi_1+\varphi_2)\sin 2\theta\ b_4 + \cos(\varphi_1+\varphi_2)\sin 2\theta\ b_5\\
\ell^4 &= \sin 2\theta \sin(\varphi_1-\varphi_2)\, b_3 + (\cos(\varphi_1+\varphi_2)\cos(\varphi_1-\varphi_2) - \sin(\varphi_1+\varphi_2)\cos 2\theta\sin(\varphi_1-\varphi_2))b_4\\
&\quad -(\sin(\varphi_1+\varphi_2)\cos(\varphi_1-\varphi_2) + \cos(\varphi_1+\varphi_2)\cos 2\theta\sin(\varphi_1-\varphi_2))\ b_5\\
\ell^5 &= -\sin 2\theta\cos(\varphi_1-\varphi_2)\, b_3 + (\cos(\varphi_1+\varphi_2)\sin(\varphi_1-\varphi_2) + \sin(\varphi_1\varphi_2)\cos 2\theta\cos(\varphi_1+\varphi_2))\, b_4\\
&\quad +(\cos(\varphi_1+\varphi_2)\cos 2\theta\cos(\varphi_1-\varphi_2) - \sin(\varphi_1+\varphi_2)\sin(\varphi_1-\varphi_2))\ b_5\,.
\end{aligned} \tag{A.34}$$

A.3 The action

Once the f's and b's are calculated, then, using the explicit components of the matrix $\mathcal{M}$ to simplify the fermionic currents, the kinetic part of the Lagrangian (up to the overall constants in (2.2)) can be written in the form

$$\mathcal{L} = \sum_{m,n=0}^{5} \eta_{mn}(\ell+j)^m(\ell+j)^n + \sum_{a,b,\alpha,\beta} \epsilon_{\alpha\beta}\delta_{ab} f^{a\alpha} f^{b\beta}\,. \tag{A.35}$$

All the currents have been explicitly written out above and, one can write out the full action to all orders in the fermionic coordinates.

Normalization. In order to check the normalization, it is useful to focus on the purely bosonic part. Let us first write the worldsheet action explicitly in terms of the Poincaré coordinates $(\gamma,\bar\gamma,\phi)$. Taking into account $\eta^{z\bar z}=2$ and rewriting the partial derivatives of g^{-1} in terms of that on g, we get

$$S_{\rm WS} = \frac{1}{4\pi f^2}\int d^2z\, {\rm STr}(g^{-1}\partial g\, g^{-1}\bar\partial g) + S_{\rm WZW}\,. \tag{A.36}$$

Using equation (A.22) and combining this with an antisymmetric part coming from the WZW term, we find that

$$\begin{aligned}
S_{\rm WS} &= \frac{1}{4\pi}\int d^2z\left\{e^{2\phi}\left[\left(\frac{1}{f^2}+k\right)\partial\bar\gamma\bar\partial\gamma + \left(\frac{1}{f^2}-k\right)\partial\gamma\bar\partial\bar\gamma\right] + \frac{2}{f^2}\partial\phi\bar\partial\phi\right\} + \dots\\
&= \frac{1}{2\pi}\int d^2z\left(e^{2\phi}\left[-c_+\partial\bar\gamma\bar\partial\gamma - c_-\partial\gamma\bar\partial\bar\gamma\right] + \frac{1}{f^2}\partial\phi\bar\partial\phi\right) + \dots\,.
\end{aligned} \tag{A.37}$$

One can check that, at the WZW point, putting $f^2=1/k$, the action coincides with the bosonic action on AdS_3 with NS-NS flux with an overall multiplicative factor of $k/2\pi$.

B Primary operators

We define a primary field ϕ with respect to the left-current algebra (3.4) as a field satisfying the OPEs:

$$j^a_{L,z}(z,\bar{z})\phi(w,\bar{w}) = -\frac{c_+}{c_+ + c_-}\frac{t^a\phi(w,\bar{w})}{(z-w)} + \text{less singular} \tag{B.1}$$

$$j^a_{L,\bar{z}}(z,\bar{z})\phi(w,\bar{w}) = -\frac{c_-}{c_+ + c_-}\frac{t^a\phi(w,\bar{w})}{(z-w)} + \text{less singular} \tag{B.2}$$

where t^a is a generator of the Lie super-algebra in the representation in which ϕ transforms on the left. The structure of these OPEs is postulated, and the exact values of the coefficients is fixed by demanding compatibility both with current conservation and with the Maurer-Cartan equation. We will now show that this definition implies that a primary field with respect to the left-current algebra is also a primary field with respect to the Virasoro algebra. The worldsheet stress tensor is:[8]

$$T(z) = \frac{1}{2c_1}\kappa_{ab} : j^a_{L,z} j^b_{L,z} : (z). \tag{B.3}$$

Let us compute the OPE between a left-primary field ϕ and the holomorphic stress-tensor:

$$\begin{aligned}
\phi(z)T(w) &= \frac{1}{2c_1}\lim_{:x\to w:}\phi(z) j_{aL,z}(x) j^a_{L,z}(w)\\
&= \frac{1}{2c_1}\lim_{:x\to w:}\left(-\frac{c_+}{c_+ + c_-}\frac{t_a\phi(x)}{x-z} j^a_{L,z}(w) - \frac{c_+}{c_+ + c_-} j^a_{L,z}(x)\frac{t_a\phi(w)}{w-z}\right)\\
&= \frac{c_+}{2c_1(c_+ + c_-)}\lim_{:x\to w:}\left(\frac{t_a}{x-z}\left(\frac{c_+}{c_+ + c_-}\frac{t^a\phi(w)}{w-x} - :\phi j^a_{L,z}:(w)\right)\right.\\
&\qquad\qquad \left.+\frac{t_a}{w-z}\left(\frac{c_+}{c_+ + c_-}\frac{t^a\phi(w)}{x-w} - :j^a_{L,z}\phi:(w)\right)\right)\\
&= \frac{c_+}{2c_1(c_+ + c_-)}\left(\frac{c_+}{c_+ + c_-}\frac{t_a t^a\phi(w)}{(z-w)^2} - 2\frac{t_a : j^a_{L,z}\phi:(w)}{w-z} - \frac{c_+}{c_+ + c_-}\frac{t_a t^a\partial\phi(w)}{w-z}\right)\\
&= \frac{f^2}{2}\frac{t_a t^a\phi(w)}{(z-w)^2} + \frac{1}{c_+}\frac{t_a : j^a_{L,z}\phi:(w)}{w-z} - \frac{f^2}{2}\frac{t_a t^a\partial\phi(w)}{w-z}.
\end{aligned} \tag{B.4}$$

We deduce the OPE between the stress-tensor and the primary field ϕ:

$$T(w)\phi(z) = \frac{f^2}{2}\frac{t_a t^a\phi(z)}{(w-z)^2} + \frac{1}{c_+}\frac{t_a : j^a_{L,z}\phi:(z)}{w-z}. \tag{B.5}$$

We observe that there is no pole of order greater than two, and that the pole of order two is proportional to the operator ϕ. This implies that the operator ϕ is annihilated by all the positive modes of the stress tensor, and thus that it is a Virasoro primary. We can read off the conformal dimension of the operator ϕ:

$$\Delta_\phi = \frac{f^2}{2} t_a t^a. \tag{B.6}$$

The simple pole in the $T.\phi$ OPE gives the holomorphic derivative of ϕ:

$$\partial\phi = \frac{1}{c_+} t_a : j^a_{L,z}\phi:(z). \tag{B.7}$$

[8] We will not be careful about minus signs due to fermionic statistics. They can be consistently restored.

C The Virasoro algebra

In this appendix we will consider the self-OPE of the stress tensor (5.7). As we did in the bulk of the paper, we will evaluate the OPE of the stress-tensor with its anti-holomorphic derivative :

$$\begin{aligned}\partial_{\bar{x}}T(x)\cdot T(y) =& -\frac{1}{4\pi i}\int d^2w \\ &\times\left[\oint dz(\partial_x j_{L,z}\partial_x\Phi_1+2\partial_x^2 j_{L,z}\Phi_1)+\oint d\bar{z}(\partial_x j_{L,\bar{z}}\partial_x\Phi_1+2\partial_x^2 j_{L,\bar{z}}\Phi_1)\right] \\ &\times\left[(\partial_y j_{L,z}\partial_y\partial_{\bar{w}}\Lambda+2\partial_y^2 j_{L,z}\partial_{\bar{w}}\Lambda)+(\partial_y j_{L,\bar{z}}\partial_y\partial_{\bar{w}}\Lambda+2\partial_y^2 j_{L,\bar{z}}\partial_{\bar{w}}\Lambda)\right] \end{aligned} \tag{C.1}$$

where we kept the coordinate dependence of the operators implicit. In order to compute this OPE, we require the OPEs between the x-dependent combinations of the currents (and their derivatives), which we list below for convenience:

$$\begin{aligned}
\partial_x j_{L,z}\cdot\partial_y j_{L,z} &\sim \frac{-2c_1}{(z-w)^2}+\frac{2c_2(x-y)\partial_y^2 j_{L,z}}{i(z-w)}+\frac{2(c_2-g)(\bar{z}-\bar{w})(x-y)\partial_y^2 j_{L,\bar{z}}}{i(z-w)^2}\\
\partial_x j_{L,z}\cdot\partial_y^2 j_{L,z} &\sim \frac{-2c_2\partial_y^2 j_{L,z}}{i(z-w)}+\frac{-2(c_2-g)(\bar{z}-\bar{w})\partial_y^2 j_{L,\bar{z}}}{i(z-w)^2}\\
\partial_x j_{L,z}\cdot\partial_y j_{L,\bar{z}} &\sim -2\tilde{c}\delta^{(2)}(z-w)+\frac{2(c_4-g)f_c^{ab}(x-y)\partial_y^2 j_{L,z}}{i(\bar{z}-\bar{w})}+\frac{2(c_2-g)f_c^{ab}(x-y)\partial_y^2 j_{L,\bar{z}}}{i(z-w)}\\
\partial_x j_{L,z}\cdot\partial_y^2 j_{L,\bar{z}} &\sim \frac{-2(c_4-g)f_c^{ab}\partial_y^2 j_{L,z}}{i(\bar{z}-\bar{w})}+\frac{-2(c_2-g)f_c^{ab}\partial_y^2 j_{L,\bar{z}}}{i(z-w)}\\
\partial_x^2 j_{L,z}\cdot\partial_y j_{L,z} &\sim \frac{2c_2\partial_y^2 j_{L,z}}{i(z-w)}+\frac{2(c_2-g)(\bar{z}-\bar{w})\partial_y^2 j_{L,\bar{z}}}{i(z-w)^2}\\
\partial_x^2 j_{L,z}\cdot\partial_y j_{L,\bar{z}} &\sim \frac{2(c_4-g)f_c^{ab}\partial_y^2 j_{L,z}}{i(\bar{z}-\bar{w})}+\frac{2(c_2-g)f_c^{ab}\partial_y^2 j_{L,\bar{z}}}{i(z-w)}\\
\partial_x j_{L,z}\Phi_1 &\sim \frac{c_+}{c_++c_-}\frac{1}{z-w}[2(x-y)\partial_y-2]\Phi_1\\
\partial_x j_{L,z}\partial_y\Phi_1 &\sim \frac{c_+}{c_++c_-}\frac{1}{z-w}[2(x-y)\partial_y^2-4\partial_y]\Phi_1\\
\partial_x^2 j_{L,z}\Phi_1 &\sim \frac{c_+}{c_++c_-}\frac{2\partial_y\Phi_1}{z-w}\\
\partial_x^2 j_{L,z}\partial_y\Phi_1 &\sim \frac{c_+}{c_++c_-}\frac{2\partial_y^2\Phi_1}{z-w}\,.
\end{aligned} \tag{C.2}$$

The OPEs of the current component $j_{L,\bar{z}}$ with the operator Φ_1 are identical except the overall factor outside is $c_-/(c_++c_-)$ and the antiholomorphic factor $1/(\bar{z}-\bar{w})$ replaces the holomorphic factor. The contour integrals in equation (C.1) will pick up the poles in the OPEs between the integrated operators. One useful point to note is that the mixed terms in the OPEs, which contain both holomorphic and anti-holomorphic pieces in $z-w$ do not contribute to the contour integrals. The full computation is quite tedious. The key point is that because of the relations (3.6) satisfied by the coefficients of the current algebra, the computation is a simple non-chiral generalization of the calculation for the case with pure NS flux [10].

Let us illustrate this for the most singular term in the above OPE. It is obtained by taking the most singular terms in the current-current OPEs (i.e. the doubles poles and the contact terms). Collecting the four relevant terms in equation (C.1), we get :

$$\begin{aligned}&-\frac{1}{4\pi i}\int d^2w \oint dz \left[\frac{-2c_1}{(z-w)^2}\partial_x\Phi_1\cdot\partial_y\partial_{\bar w}\Lambda - 2\tilde c\delta^{(2)}(z-w)\partial_x\Phi_1\cdot\partial_y\partial_w\Lambda\right]\\ &-\frac{1}{4\pi i}\int d^2w \oint d\bar z \left[\frac{-2c_3}{(\bar z-\bar w)^2}\partial_x\Phi_1\cdot\partial_y\partial_w\Lambda - 2\tilde c\delta^{(2)}(z-w)\partial_x\Phi_1\cdot\partial_y\partial_{\bar w}\Lambda\right]. \quad (C.3)\end{aligned}$$

Let us first combine the first and the fourth terms, followed by the second and the third terms. After performing the contour integration, this leads to

$$\begin{aligned}\int d^2w \Bigg[&\frac{(c_1-\tilde c)}{c_+}\lim_{:z\to w:}\left[\partial_x^2(j_{L,z}\Phi_1)(x;w)\cdot\partial_y\partial_{\bar w}\Lambda(y;w)\right]\\ &+\frac{(c_3-\tilde c)}{c_-}\lim_{:z\to w:}\left[\partial_x^2(j_{L,\bar z}\Phi_1)(x;w)\cdot\partial_y\partial_w\Lambda(y;w)\right]\Bigg] \quad (C.4)\end{aligned}$$

The coefficients simplify thanks to the relations (3.6). The regular limit leads to a $\delta^{(2)}(x-y)$ factor and the most singular term in the OPE (C.1) occurs when both partial derivatives ∂_x^2 act on it. Finally we obtain :

$$\partial_{\bar x}T(x)\cdot T(y) = -\pi\partial_x^3\delta^{(2)}(x-y)I+\dots, \quad (C.5)$$

where I is the operator defined in equation (4.36) that we already encountered in the self-OPE of the R-current.

Integrating with respect to $\bar x$, we find

$$T(x)\cdot T(y) = \frac{3I}{(x-y)^4}+\dots \quad (C.6)$$

which gives $c = 6\,I$. This generalizes the result of [10] to $AdS_3\times S^3$ backgrounds which include RR fluxes. A similar analysis can be done for all the other terms leading to the standard operator product expansion for the energy-momentum tensor.

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